1.4: Separable Equations

Definition 1. A first-order differential equation $dy/dx = f(x, y)$ is **separable** if the function f can be written as the product of two independent functions of one variable each; i.e. $f(x, y) = g(x)h(y)$. In this case, we simply separate the y's and x's and integrate them separately.

Example 1. Find the general solution for the differential equation

$$\frac{dy}{dx} = -6xy.$$

Example 2. Solve the differential equation

$$\frac{dy}{dx} = \frac{4 - 2x}{3y^2 - 5}.$$

Exercise 1. Find all solutions to the differential equation

$$\frac{dy}{dx} = 6x(y-1)^{2/3}.$$

Exercise 2. Find all solutions to the differential equation

$$2\sqrt{x}\frac{dy}{dx} = \cos^2 y, \quad y(4) = \pi/4.$$

Natural Growth and Decay. The differential equation

$$\frac{dx}{dt} = kx \quad (k \text{ is a constant}) \tag{1}$$

serves as a mathematical model for a wide variety of natural phenomena, such as, population growth, compound interest, radioactive decay or drug elimination to name a few. Solve the differential equation given in (1).

Example 3. A specimen of charcoal found at Stonehenge turns out to contain 63% as much ^{14}C as a sample of present-day charcoal of equal mass. Given that the half-life of ^{14}C is 5700 years, we can solve for the constant $k \approx 0.0001216$. What is the age of the sample?

Example 4. Recall that Newton's law of cooling is given by the differential equation

$$\frac{dT}{dt} = k(A - T)$$

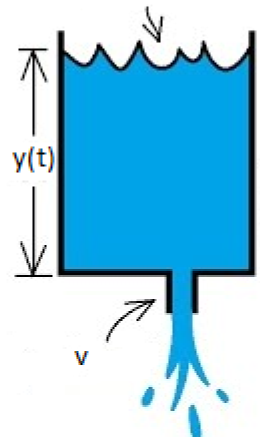
for a positive constant k, where $T(t)$ is the temperature of a body immersed in a medium of constant temperature A. Consider the following: A 4-lb roast, initially at 50°F, is placed in a 375°F oven at 5:00pm. After 75 minutes it is found that the temperature $T(t)$ of the roast is 125°F. When will the roast be 150°F (medium rare)?

Torricelli's Law. Suppose that a water tank has a hole with area a at the bottom. Let $y(t)$ represent the depth of water and $V(t)$ represent the volume of water in the tank. It is true, under ideal conditions, that the velocity of water exiting through the hole is

$$v = c\sqrt{2gy}.$$

(We take $c = 1$ for simplicity.) We arrive at the equation

$$\frac{dV}{dt} = -av \quad \text{or equivalently} \quad A(y)\frac{dy}{dt} = -av.$$



Exercise 3. A hemispherical bowl has to radius 4 ft and at time $t = 0$ is full of water. At that moment a circular hole with diameter 1 in. is opened in the bottom of the tank. How long will it take for all the water to drain from the tank?

Homework. 1-15, 19-23, 33-45, 49-61 (odd)